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BOSTON UNIVERSITY

GRADUATE SCHOOL

THESIS

PEARSON-----TYPE III
GENERALIZED FREQUENCY CURVE.

by

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(S.B. Boston University, 1941)

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During 1895-1916 Karl Pearson published papers in which he showed that a set of frequency curves could be obtained by assigning values to the parameters in a certain first order differential equation.

(The Pearson school claims that all the different types of frequency distributions that arise in practical statistics can be represented by the solutions of this equation.)

In the Annals of Mathematical Statistics, May, 1930, Volume 1, Number 2, there is an article, Tables of Pearson's Type III Function, by L.R. Salvosa. These tables give values for a skewness up through 1.1 in the type III curve. The original work of this paper consists in the compiling of the tables for the skewnesses: 1.3, 1.5, 1.7, 1.9,

In the material following, the symbols should be of a constant nature. This I did not find true in the several references that I shall refer to. I employed those symbols which statisticians of the present day seem to follow in their mathematical writing.

Differential Equation of the Pearson Frequency Curves.

Considering the most obvious characteristics of frequency distributions, we find that they:

1. generally start at zero,
2. rise to a maximum,
3. then fall, sometimes with symmetry but often without symmetry.
4. At the ends of the distribution there is often high contact.

This means, mathematically, that a series of equations $y=f(t)$, $y=\phi(t)$, etc, must be chosen, so that in each equation of the series $dy/dt=0$, for certain values of t, namely, at the maximum and at the end of the curve where there is contact with the axis of t.

If we consider the differential equation $dy/dt=\frac{(a-t)y}{F(t)}$, then if:

1. $y=0$, $dy/dt=0$, and there is therefore contact at the end of the curve, while if
2. $t=a$, $dy/dt=0$, and we have the maximum required.
3. If we assume that F(t) converges so rapidly that terms involving the third and fourth and higher powers of t may be neglected, then upon expanding F(t) in ascending powers of t by Maclaurin's series, we have:

$$(1) \qquad dy/dt=\frac{(a-t)y}{b_o+b_1t+b_2t^2}$$

We shall return to this equation and show how it can be put in the form $y=f(t)$, so as to express y as a direct function of t, and we shall see that we have obtained curves taking various widely different shapes.

Differential Equation of the Pearson Frequency Curves.

Considering the most obvious characteristics of frequency distributions, we find them to be:

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3. then fall, sometimes with symmetry but often without symmetry.
4. At the ends of the distribution there is often high contact.

This means, mathematically, that a series of equations $y=f(x)$, $y=\phi(x)$, etc. must be chosen, so that in each equation of the series $dy/dx=0$, for certain values of x, namely, at the maximum and at the end of the curve where there is contact with the axis of x.

If we consider the differential equation $dy/dx=\frac{(a-x)y}{f(x)}$, then if:

1. $y=0$, $dy/dx=0$, and there is therefore contact at the end of the curve, while if
2. $x=a$, $dy/dx=0$, and we have the maximum required.
3. If we assume that $f(x)$ converges so rapidly that terms involving the third and fourth and higher powers of x may be neglected, then upon expanding $f(x)$ in ascending powers of x by Maclaurin's series, we have:

$$(1) \qquad dy/dx=\frac{(a-x)y}{c+bx+bx^2}$$

If small return to this equation and upon how it can be put in the form $y=f(x)$, so as to express y as a direct function of x, and we shall see that we have obtained curves taking various wholly different shapes.

Genesis of the Pearson Frequency Curves in the Theory of Probability.

We can associate (1) with underlying probabilities by considering the following problem:

> An urn contains n balls of which np are white, so that the probability of drawing a white ball in a single trial is p. The rest of the balls, nq are black, and the probability of failure to draw a white ball in a single trial is q = 1-p. If N balls are drawn from the urn one at a time with replacements after each draw, what is the probability, B(k), of drawing exactly k white balls and (N-k) black balls?

From the Bernoulli (Binomial distribution) theory, it is known that the probabilities of getting k = 0,1,2,.....N successes in N trials are given by the successive terms of the binomial

$$(q+p)^N = \sum_{K=0}^{N} B(k) \qquad \text{where } B(k) = \frac{N!}{k!(N-k)!} p^k q^{N-k}$$

Representing the terms B(k) by ordinates y_N, one may plot the (N+1) points (k, y_k). Through these (N+1) points one may imagine a curve that can be represented by an analytic function.

Since: $$y_K = \frac{N!}{K!(N-K)!} p^K q^{N-K}$$

And hence: $$y_{K+1} = \frac{N!}{(K+1)!(N-K+1)!} p^{K+1} q^{N-K-1}$$

We have:

$$\frac{y_{K+1}}{y_K} = \frac{N! p^K \cdot p^1 \cdot q^N \cdot q^{-K} \cdot q^{-1} \cdot K!(N-K)!}{(K+1)!(N-K+1)! N! p^K \cdot q^N \cdot q^{-K}} = \frac{p(N-K)}{q(K+1)}$$

$$= \frac{Np - pK}{qK + q} \qquad (a)$$

From (a) we have:

$$(b)\quad \frac{y_{K+1}-y_K}{\frac{1}{2}(y_{K+1}+y_K)} = \frac{Np-pK-qK-q}{Np-pK+qK+q} = \frac{Np-K(p+q)-q}{Np+q+K(q-p)}$$

Now the mean of any two ordinates (y_K and y_{K+1}) may be considered as approximately equal to the ordinate ($y_{K+\frac{1}{2}}$) midway between them.

The slope of the line joining any two points (K, y_K) and $(K+1, y_{K+1})$ is also approximately equal to the slope of the tangent at the point midway between these two points on the continuous curve.

Under these two assumptions (b) may be written as:

$$\frac{D_K\, y_{K+\frac{1}{2}}}{y_{K+\frac{1}{2}}} = \frac{2(Np-q-K)}{Np+q+K(q-p)}$$

The right-hand member of this equation is, therefore, the derivative of log y at the point $(K+\frac{1}{2}, y_{K+\frac{1}{2}})$. At any point (K, y_K) this derivative is:

$$(c)\quad \frac{d(\log y)}{dK} = \frac{2\{Np-q-(K-\frac{1}{2})\}}{Np+q+(K-\frac{1}{2})(q-p)}$$

Case 1. $p = q = \frac{1}{2}$, then (c) becomes

$$\frac{d}{dK}(\log y) = \frac{-(K-N/2)}{(\frac{N+1}{4})}$$ which is of the form

$$\frac{dy}{y} = -\frac{(K-a)\,dK}{b_0} \quad b_0>0$$ and which upon integration yields the normal curve.

Case 2. $p \neq q$, then from (c) we have

$$(d) \quad \frac{d}{dK}(\log y_x) = -\left(\frac{\frac{q-p}{2} + (K-Np)}{Npq + \frac{1}{4} + \frac{(K-Np)(q-p)}{2}}\right)$$

If we set

$$\alpha_3 = \frac{q-p}{(Npq)^{1/2}}, \quad t = \frac{K-Np}{(Npq)^{1/2}}$$ then (d) becomes

$$(e) \quad \frac{d}{dt}(\log y_x) = \frac{\alpha_3/2 + t}{1 + \alpha_3/2\, t}$$

If Npq is so large that $\frac{1}{Npq}$ is negligible, then (e) becomes

$$(f) \quad \frac{d}{dt}(\log y_x) = -\left(\frac{\alpha_3/2 + t}{1 + \alpha_3/2\, t}\right)$$

which upon integration, yields the Type III curve.

Special Cases:

With $p = q$, (c) is of form (1) when $b_1 = b_2 = 0$.

$p \neq q$, (c) is of form (1) when $b_2 = 0$.

The parameters in (1) can be expressed in terms of the moments of the system.

$$\frac{dy}{dt}(b_0 + b_1 t + b_2 t^2) = (a - t)y$$

Multiply by $t^n dt$ and integrate over all admissible values of t.

$$\int \frac{dy}{dt}(b_0 t^n + b_1 t^{n+1} + b_2 t^{n+2})\,dt = \int y\,(at^n - t^{n+1})\,dt$$

Integrating the left-hand side by parts:

$$\text{(g)}\quad t^n(b_0 + b_1 t + b_2 t^2)y - \int y\,[b_0 n t^{n-1} + b_1(n+1)t^n + b_2(n+2)t^{n+1}]\,dt = \int y\,(at^n - t^{n+1})\,dt$$

If $y t^{n+2}$ vanishes at the ends of the range, then the first expression (g) vanishes.

If $y = h(t)$, we have from the fact that:

$$\alpha_n = \frac{\mu_n}{\sigma_n} = \int_{-\infty}^{\infty} t^n h(t)\,dt$$

and (g), that

$$\text{(h)}\quad a\alpha_n + b_0 n\alpha_{n-1} + b_1(n+1)\alpha_n + b_2(n+2)\alpha_{n+1} = \alpha_{n+1}$$

Assigning n successively the values n= 0,1,2,3, we obtain:

$$\begin{aligned} a \quad &+b_1 \quad &=0 \\ b_0 \quad &+3b_2 \quad &=1 \end{aligned}$$

(i)

$$\begin{aligned} a \quad &+3b_1 \quad &+4b_2\alpha_3 = \alpha_3 \\ a\alpha_3 + 3b_0 \quad &+4b_1\alpha_3 \quad &+5b_2\alpha_4 = \alpha_4 \end{aligned}$$

Making note of the fact that:

$\alpha_0 = 1$

$\alpha_1 = 0$

$\alpha_2 = 1$

From (i) we can determine the parameters we are interested in.

A simultaneous solution of (i) yields:

$$(2) \qquad a = -\frac{\alpha_3}{2(1+2\delta)} \qquad b_1 = \frac{\alpha_3}{2(1+2\delta)}$$

$$b_0 = \frac{2+\delta}{2(1+2\delta)} \qquad b_2 = \frac{\delta}{2(1+2\delta)}$$

where $\delta = \dfrac{2\alpha_4 - 3\alpha_3^2 - 6}{\alpha_4 + 3}$

The value of $a = -\dfrac{\alpha_3}{2(1+2\delta)}$ which represents the distance between the mode and the mean expressed in standard units, is called by Pearson the "skewness" of the distribution.

.....

It is useful to note:

1. (2) is valid except when $\delta = -1/2$.

2. $-2 < \delta < 2$

To show this, using a well-known device, we see that:

$$* \int_{-\infty}^{\infty} f(t)(t^2 + \lambda t)^2 dt = \alpha_4 + 2\lambda\alpha_3 + \lambda^2$$

Using: $\int t^n f(t)\,dt = \alpha_n = \dfrac{\mu_n}{\sigma_n}$

Then: $\int t^4 f(t)\,dt + \int 2\lambda t^3 f(t)\,dt + \int \lambda^2 t^2 f(t)\,dt =$

$$\alpha_4 + 2\lambda\alpha_3 + \lambda^2$$

Q.E.D.

.....

* is never negative since $f(t) \geq 0, -\infty \leq t \leq \infty$ for any real λ. This requires that $\alpha_3^2 \leq \alpha_4$

$$-2 + \frac{4\alpha_4 - 3\alpha_3^2}{\alpha_4 + 3} = \delta = 2 - \frac{3(\alpha_3^2 + 4)}{\alpha_4 + 3}$$

and the result follows that $-2 < \delta < 2$.

Turning now to the integration of (1) and the development of the various forms of f(t) that arise:

1. Over the range of variation of t, $f(t) \geq 0$.
2. The area under the curve $y = f(t)$ must be finite.

   This being true, we always determine the constant of integration so that this area is unity.
3. The range in each case is taken as the maximum one for which (1) and (2) may be secured which contains the point, $t = a$.
4. It is sufficient throughout to take $\alpha_3 \geq 0$ since the curve for $\alpha_3 = -K$ is only a reflection of that for $\alpha_3 = K$ through the line $t = 0$.

Turning now to the integration of (1) and the development of the various forms of $f(z)$ that arise:

1. Over the range of variation of $\underline{z}$, $f(z) \geq 0$.

2. The area under the curve $y = f(z)$ must be finite.

   This being true, we simply determine the constant of integration so that this area is unity.

3. The range in each case is taken as the maximum one for which (1) and (2) may be assumed which contains the point, $z = 0$.

4. It is sufficient thereafter to take ϕ containing the curve for $\phi = K$ is only a reflection of that for $\phi = K$ through the line $z = 0$.

TRANSITIONAL TYPE N

(NORMAL FREQUENCY CURVE)

Transitional Type: The Normal Frequency Function.

$$\frac{dy}{dt} = \frac{(a-t)y}{b_0+b_1t+b_2t^2} \qquad a = \frac{-\alpha_3}{2(1+2\delta)} \qquad b_1 = \frac{\alpha_3}{2(1+2\delta)}$$

$$b_0 = \frac{2+\delta}{2(1+2\delta)} \qquad b_2 = \frac{\delta}{2(1+2\delta)}$$

$\alpha_3 = \delta = 0$

then $a = 0$

$b_0 = 1$

$b_1 = 0$

$b_2 = 0$

Hence

$$\frac{dy}{dt} = -ty$$

Integration:

$$\int \frac{dy}{y} = \int -t\,dt$$

$$\log y = -t^2/2 + \log c$$

$$\log y/c = -t^2/2$$

$$\frac{y}{c} = e^{-t^2/2}$$

$$y = ce^{-t^2/2} \qquad -\infty < t < \infty$$

Constant:

$$c = (2\pi)^{-1/2}$$

NORMAL CURVE: $y = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$ Standard form

Chart: Single-point (N).

(Refer to pages 35, 36.)

TRANSITIONAL TYPE 111

Transitional Type III

$$\frac{dy}{dt} = \frac{(a-t)y}{b_0+b_1t+b_2t^2} \qquad a = \frac{-\alpha_3}{2(1+2\delta)} \qquad b_1 = \frac{\alpha_3}{2(1+2\delta)}$$

$$\alpha_3 \neq 0\ ;\ \delta = 0 \qquad b_0 = \frac{2+\delta}{2(1+2\delta)} \qquad b_2 = \frac{\delta}{2(1+2\delta)}$$

Integration: $\frac{1}{y}\frac{dy}{dt} = \frac{-\frac{\alpha_3}{2} - t}{1+\frac{\alpha_3}{2}t}$

Let $A = \frac{2}{\alpha_3}$; then $\frac{1}{A} = \frac{\alpha_3}{2}$

$$\frac{dy}{y} = \frac{(-\frac{1}{A}-t)dt}{1+t/A} = \frac{-\left(\frac{1+At}{A}\right)dt}{\frac{A+t}{A}} = \frac{-(1+At)\,dt}{(A+t)}$$

$$\log y = -\int\frac{dt}{A+t} - A\int\frac{At\,dt}{A+t}$$

$$= -\log(A+t) - A[A+t-A\log(A+t)] + \log c'$$

$$= -\log(A+t) - A^2 - At + A^2\log(A+t) + \log c'$$

$$= \log\left[\frac{(A+t)^{A^2}}{(A+t)}\right] - A(A+t) + \log c'$$

$$= \log(A+t)^{A^2-1} (+c') - A^2 - At \qquad (c'-A^2 = c)$$

$$\therefore \boxed{y = c(A+t)^{A^2-1}e^{-At}}$$

Constant: $\frac{1}{c} = \int_{-A}^{\infty}(A+t)^{A^2-1}e^{-At}\,dt$

Let $x = A+t$

$$\frac{1}{c} = \frac{e^{A^2}}{A\cdot A^{A^2-1}}\int_0^{\infty}(Ax)^{A^2-1}e^{-Ax}\,d(Ax) = \frac{e^{A^2}}{A^{A^2}}\cdot\Gamma(A^2)$$

$$\therefore \boxed{c = \frac{A^{A^2}e^{-A^2}}{\Gamma(A^2)}}$$

TYPE III $y = \frac{A^{A^2} e^{-A^2}}{\Gamma(A^2)} (A+t)^{A^2-1} e^{-At}$

Shapes of curves:

$A^2 - 1 > -1$ conditions are satisfied.

$A^2 > 1$ bell-shaped $(\alpha_3^2 < 4)$.

$A^2 < 1$ J-shaped.

Range: $(-A, \infty)$

Chart: Point falls on Line $\delta = 0$.

(Refer to pages 35, 36.)

Transitional Type X

When $A^2 = 1$, $f(t) = y = \frac{e^{-t}}{c}$ J-shaped.

Range: $(-1, \infty)$

Chart: Single point on $\delta = 0$.

(Refer to pages 35, 36.)

TABLES AND GRAPHS FOR VARIOUS SKEWNESSES OF PEARSON'S TYPE III CURVE.

For tables of values for the skewness through 1.1 , refer to:
Tables of Pearson's Type III Function by L. R. Salvosa appearing in the May, 1930 issue of the Annals of Mathematical Statistics.

The following tables are representative of the original work of this paper.

TYPE III

Skewness: $\alpha_3 = 1.3$

t	∅(t)	
-1.538	.000	
-1.400	.124	
-1.200	.309	
-1.000	.428	
-0.800	.486	
-0.650	.496	(mode)
-0.600	.495	
-0.400	.473	
-0.200	.434	
0.000	.386	
0.200	.336	
0.400	.286	
0.600	.240	
0.800	.200	
1.000	.164	
1.200	.134	
1.400	.109	
1.600	.088	
1.800	.070	
2.000	.056	
2.200	.044	
2.400	.035	
2.600	.027	
2.800	.022	
3.000	.017	
3.200	.013	
3.400	.010	
3.600	.008	
3.800	.006	
4.000	.005	
4.200	.004	
4.400	.003	
4.600	.002	
4.800	.002	
5.000	.001	

Skewness: $\alpha_3 = 1.5$

t	∅(t)	
-1.333	.000	
-1.300	.126	
-1.200	.314	
-1.100	.425	
-1.000	.493	
-0.900	.527	
-0.800	.543	
-0.750	.544	(mode)
-0.700	.542	
-0.600	.532	
-0.500	.515	
-0.400	.492	
-0.300	.467	
-0.200	.439	
-0.100	.410	
0.000	.381	
0.200	.326	
0.400	.275	
0.600	.228	
0.800	.189	
1.000	.155	
1.200	.127	
1.400	.103	
1.600	.083	
1.800	.067	
2.000	.054	
2.200	.043	
2.400	.035	
2.600	.028	
2.800	.022	
3.000	.018	
3.200	.014	
3.400	.011	
3.600	.008	
3.800	.007	
4.000	.005	
4.200	.004	
4.400	.003	
4.600	.003	
4.800	.002	
5.000	.002	

x	$\psi(x)$
-1.000	.000
-1.200	.124
-1.400	.309
-1.600	.458
-1.800	.568
-2.000	.648
-2.200	.706
-2.400	.748
-2.600	.778
-2.800	.800
-3.000	.816
-3.200	.828
-3.400	.837
-3.600	.844
-3.800	.849
-4.000	.853
-4.200	.856
-4.400	.859
-4.600	.861
-4.800	.863
-5.000	.864

Example: $\kappa_2 = 1.3$

x	$\psi(x)$
-1.000	.000
-1.200	.124
-1.400	.309
-1.600	.458
-1.800	.568
-2.000	.648
-2.200	.706
-2.400	.748
-2.600	.778
-2.800	.800
-3.000	.816
-3.200	.828
-3.400	.837
-3.600	.844
-3.800	.849
-4.000	.853
-4.200	.856
-4.400	.859
-4.600	.861
-4.800	.863
-5.000	.864

Example: $\kappa_2 = 1.3$

TYPE III

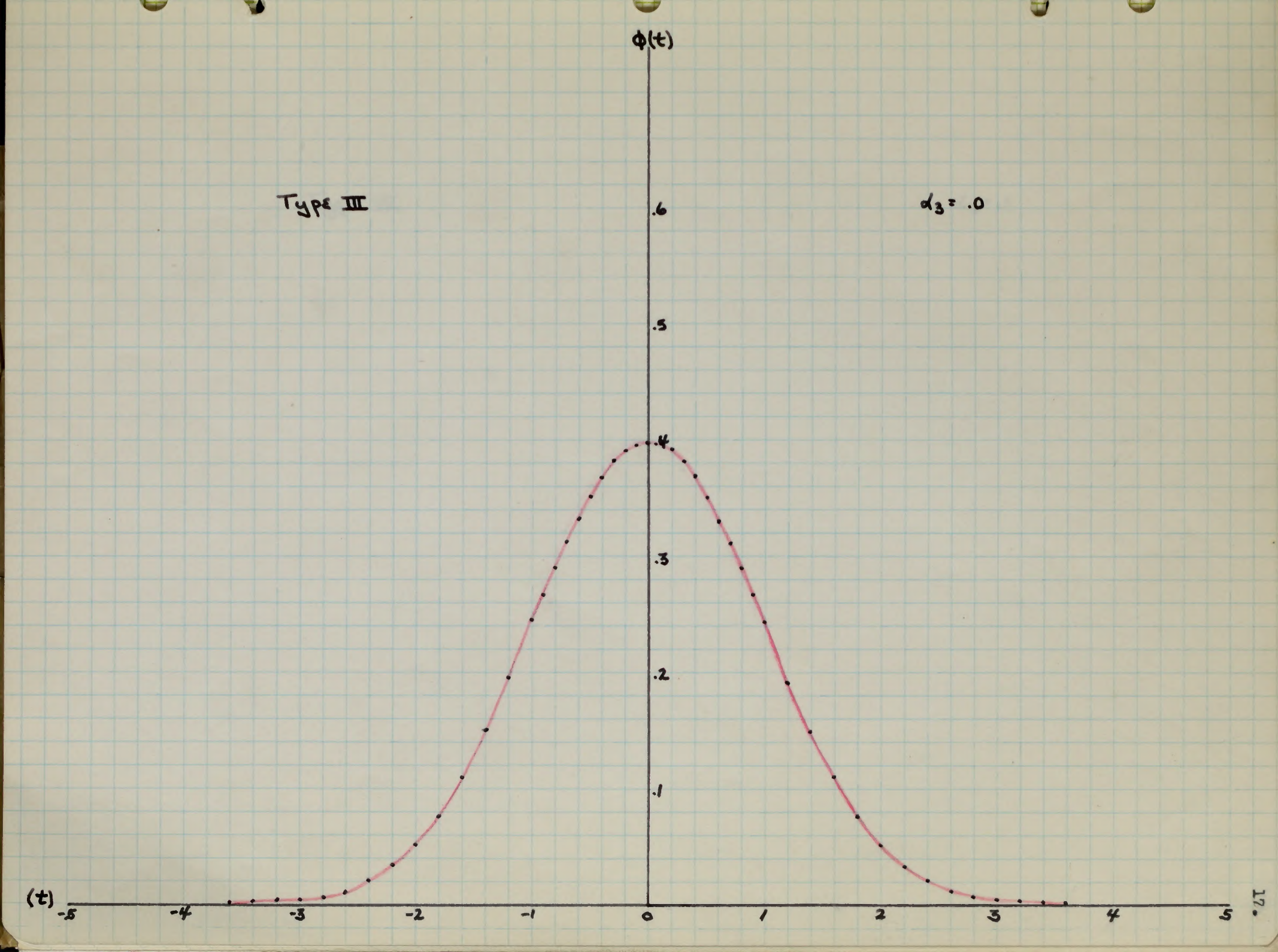
Skewness: $\alpha_3 = 1.7$

t	Ø(t)	
-1.176	.000	
-1.100	.480	
-1.000	.588	
-0.900	.621	
-0.850	.624	(mode)
-0.800	.621	
-0.700	.605	
-0.600	.578	
-0.500	.547	
-0.400	.513	
-0.300	.478	
-0.200	.442	
-0.100	.408	
0.000	.376	
0.200	.316	
0.400	.262	
0.600	.218	
0.800	.179	
1.000	.147	
1.200	.120	
1.400	.097	
1.600	.080	
1.800	.065	
2.000	.052	
2.200	.042	
2.400	.034	
2.600	.028	
2.800	.022	
3.000	.018	
3.200	.015	
3.400	.012	
3.600	.010	
3.800	.008	
4.000	.006	
4.200	.005	
4.400	.004	
4.600	.003	
4.800	.002	
5.000	.002	

Skewness: $\alpha_3 = 1.9$

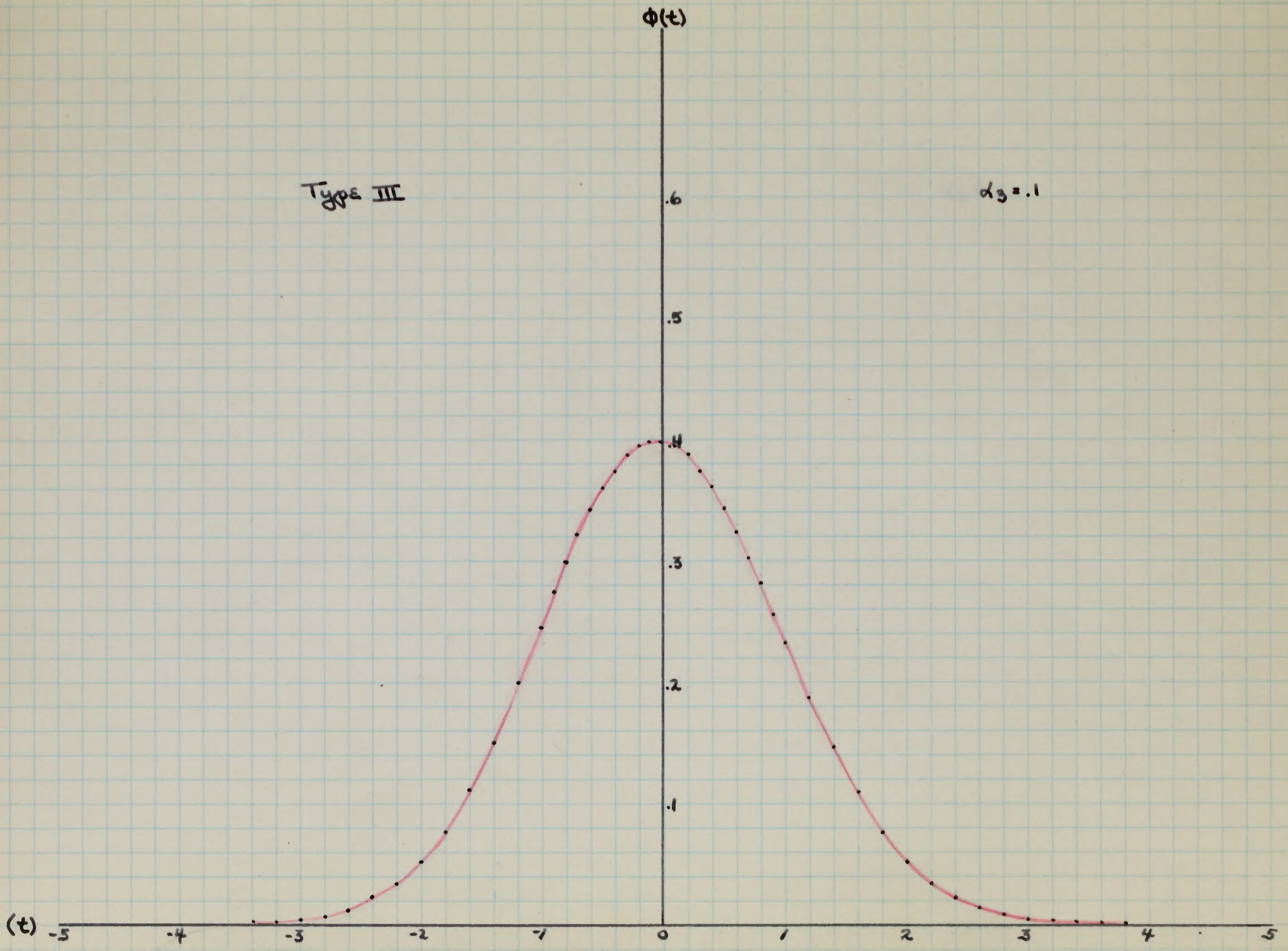
t	Ø(t)	
-1.053	.000	
-1.000	.766	
-0.950	.780	(mode)
-0.900	.773	
-0.850	.757	
-0.800	.735	
-0.700	.687	
-0.600	.635	
-0.500	.585	
-0.400	.536	
-0.300	.490	
-0.200	.448	
-0.100	.408	
0.000	.370	
0.100	.336	
0.200	.306	
0.300	.277	
0.400	.252	
0.500	.228	
0.600	.207	
0.700	.188	
0.800	.170	
0.900	.154	
1.000	.139	
1.200	.114	
1.400	.093	
1.600	.076	
1.800	.062	
2.000	.051	
2.200	.041	
2.400	.034	
2.600	.027	
2.800	.022	
3.000	.018	
3.200	.015	
3.400	.012	
3.600	.010	
3.800	.008	
4.000	.006	
4.200	.005	
4.400	.004	
4.600	.004	
4.800	.003	
5.000	.002	

t	P(t)
-1.200	.000
-1.100	.000
-1.000	.000
-0.900	.001
-0.800	.002
-0.700	.003
-0.600	.005
-0.500	.008
-0.400	.012
-0.300	.017
-0.200	.024
-0.100	.032
0.000	.042
0.100	.053
0.200	.065
0.300	.079
0.400	.094
0.500	.110
0.600	.127
0.700	.145
0.800	.163
0.900	.182
1.000	.201
1.100	.220
1.200	.239
1.300	.258
1.400	.276
1.500	.294
1.600	.311
1.700	.327
1.800	.342
1.900	.357
2.000	.370
2.100	.382
2.200	.393
2.300	.403
2.400	.412
2.500	.420
2.600	.427
2.700	.433
2.800	.438
2.900	.443
3.000	.447
3.100	.451
3.200	.454
3.300	.457
3.400	.459
3.500	.461
3.600	.463
3.700	.464
3.800	.465
3.900	.466
4.000	.467
4.100	.468
4.200	.468
4.300	.469
4.400	.469
4.500	.469
4.600	.470
4.700	.470
4.800	.470
4.900	.470
5.000	.470


Type III
$\alpha_3 = .0$
$\phi(t)$
.6
.5
.4
.3
.2
.1
(t)
-5
-4
-3
-2
-1
0
1
2
3
4
5

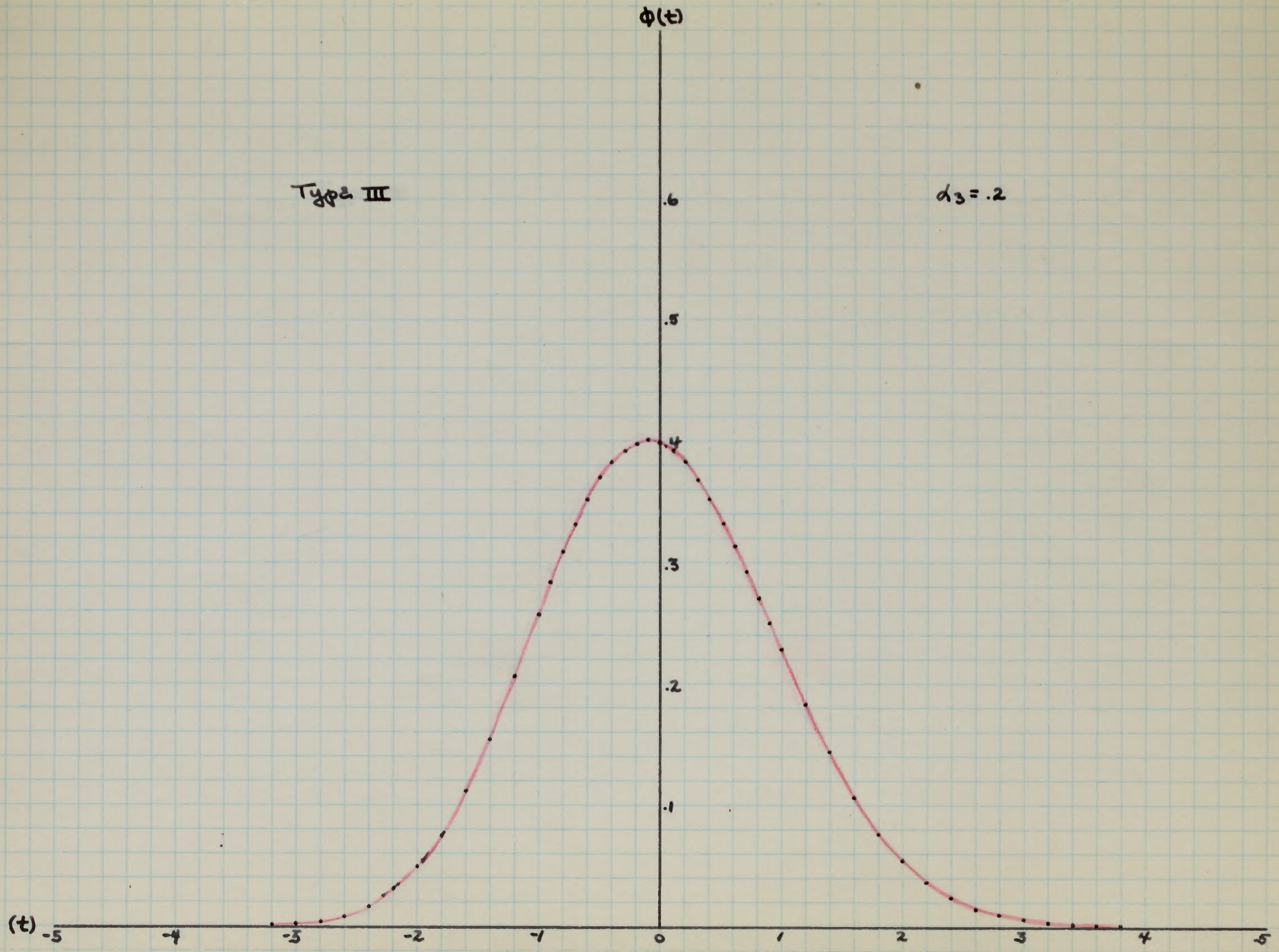
Type III

$\alpha_3 = .1$



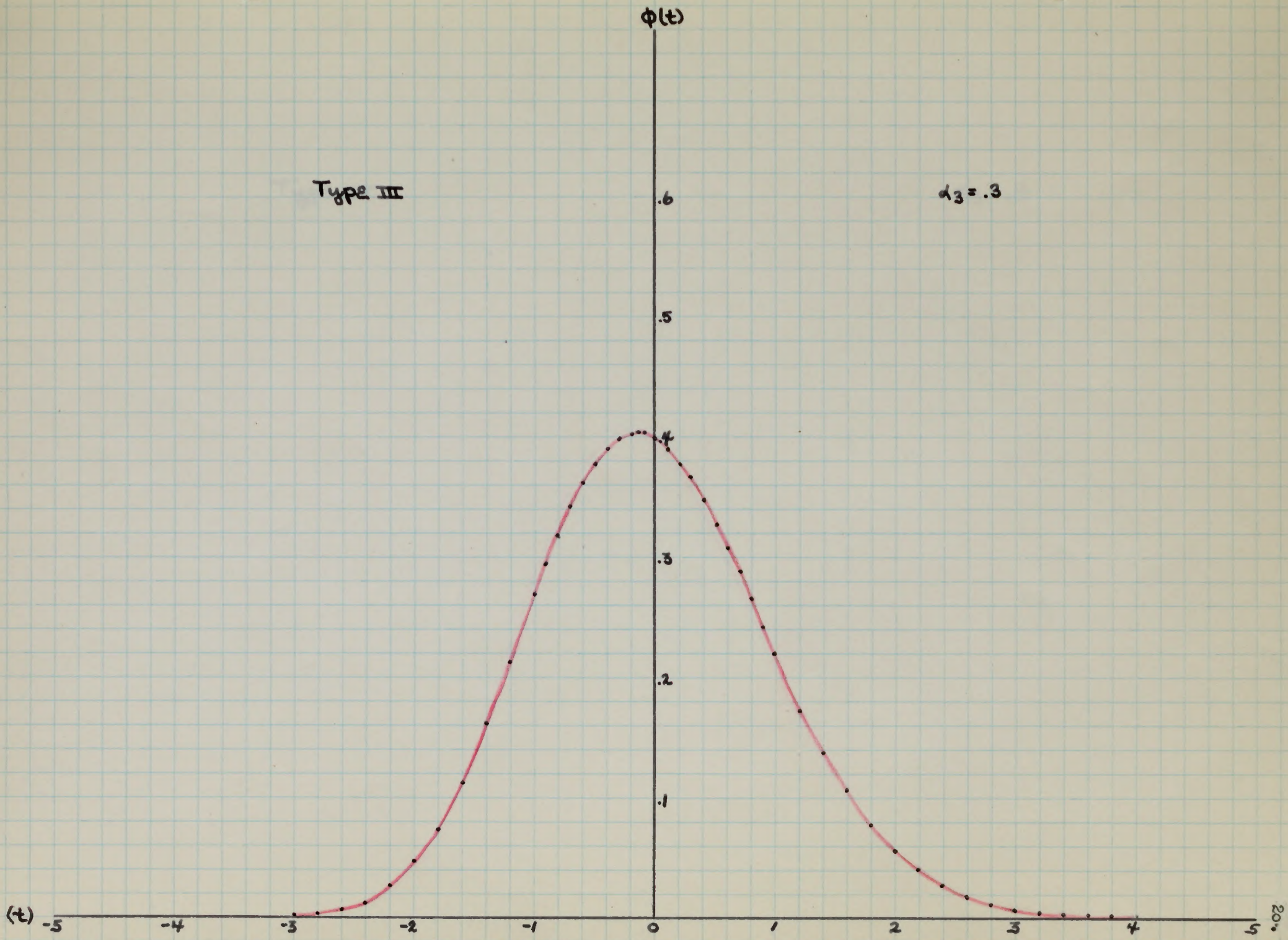
Type III

$\alpha_3 = .2$



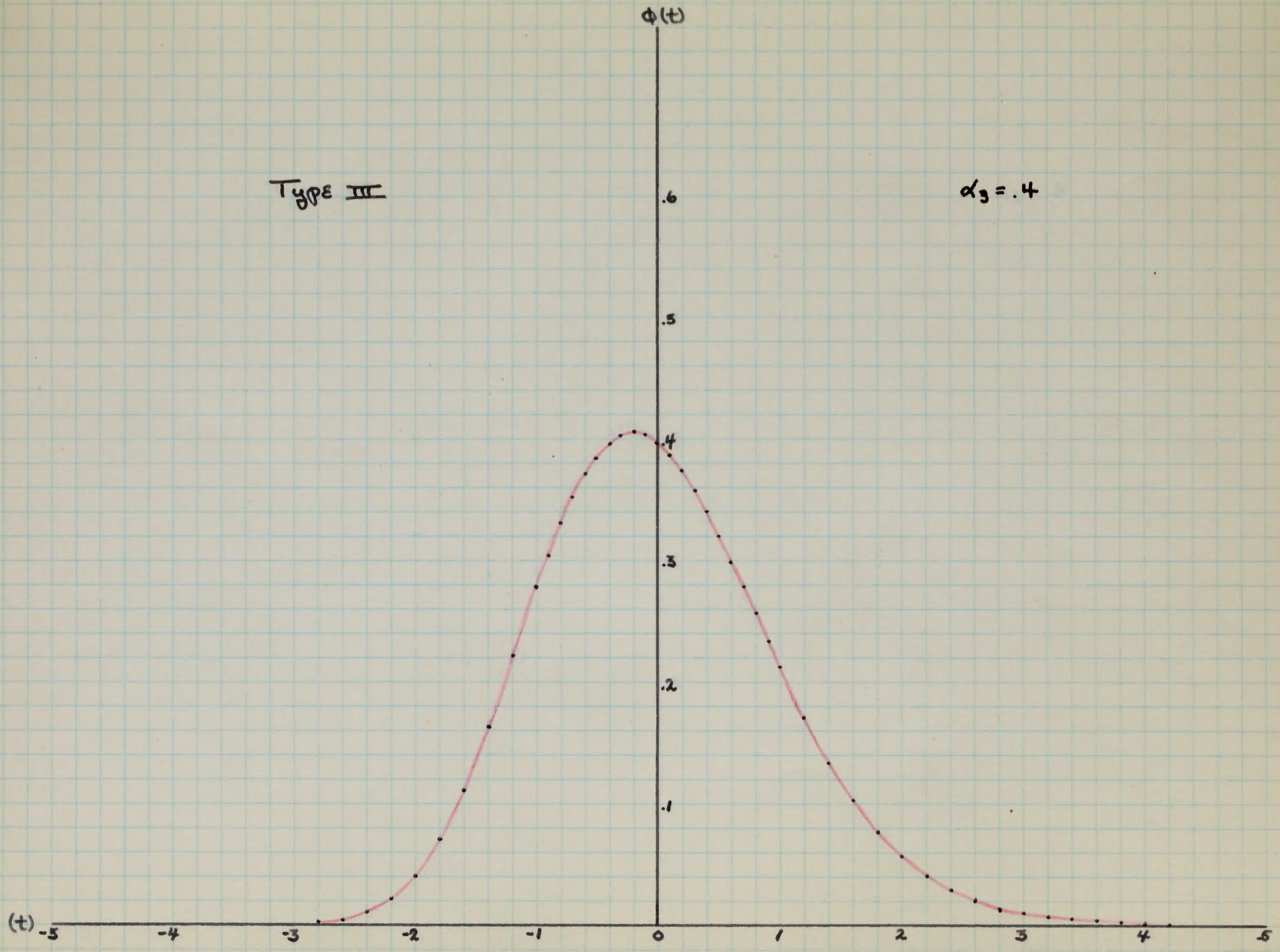
Type III

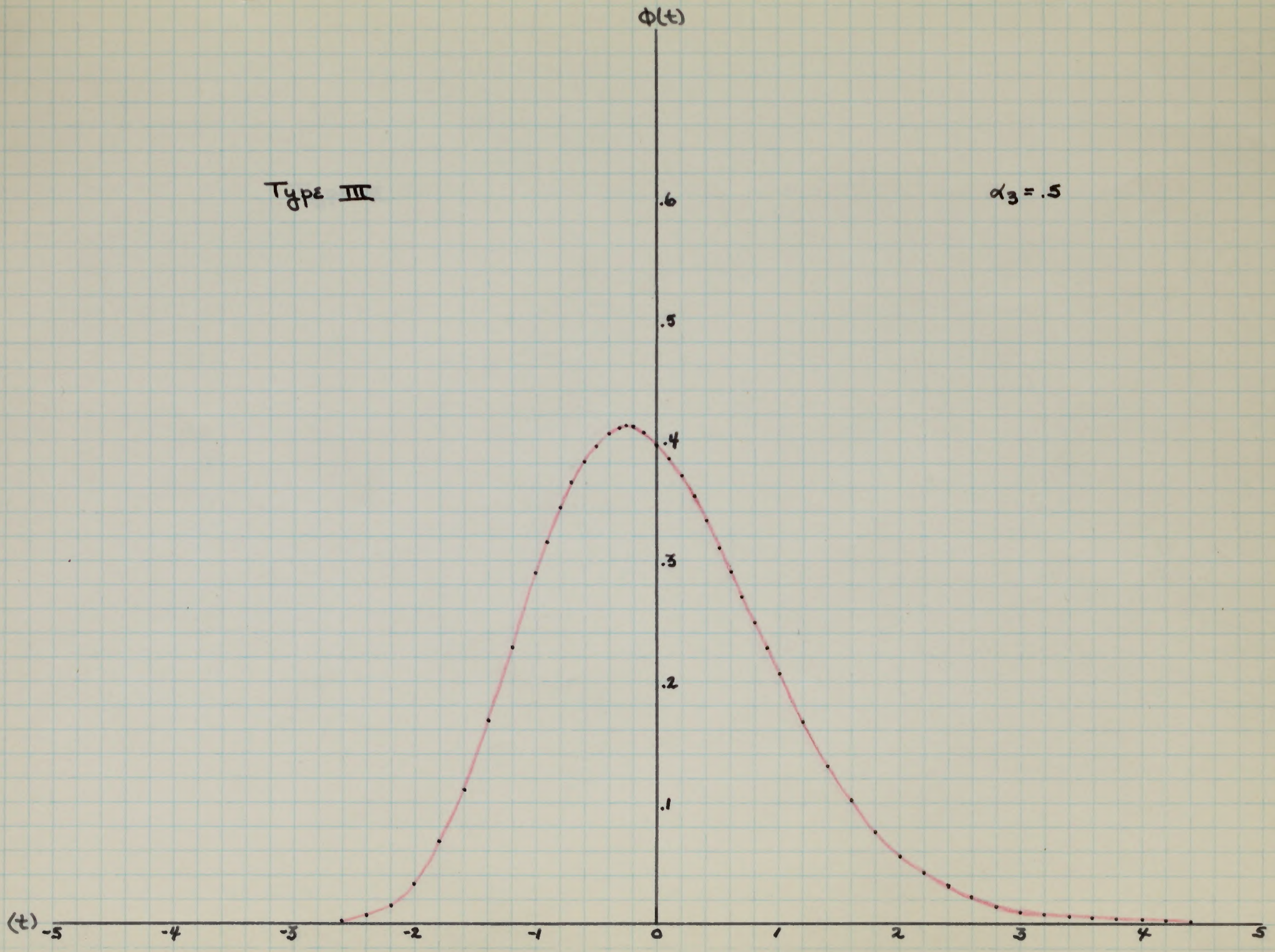
$\alpha_3 = .3$

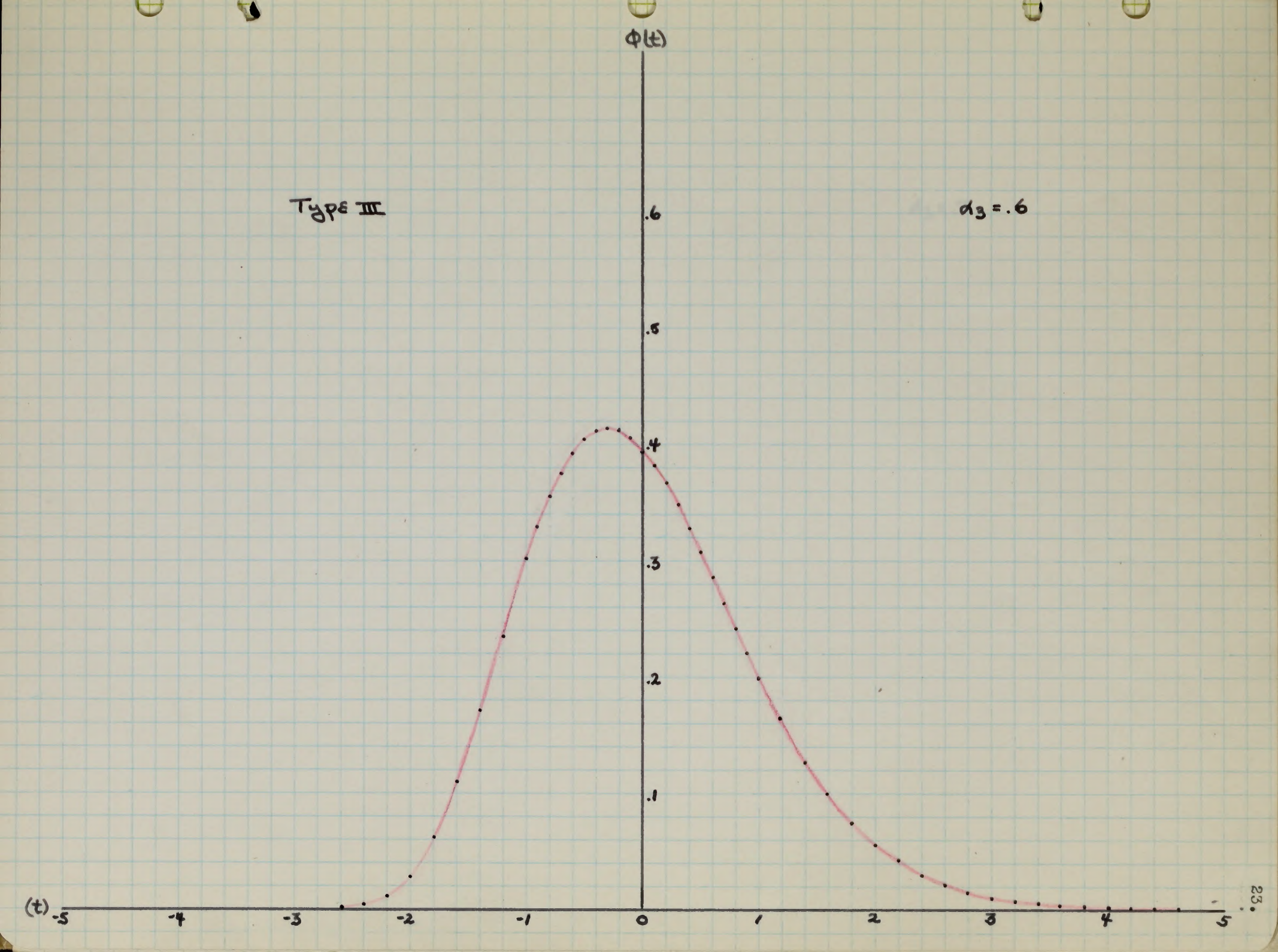


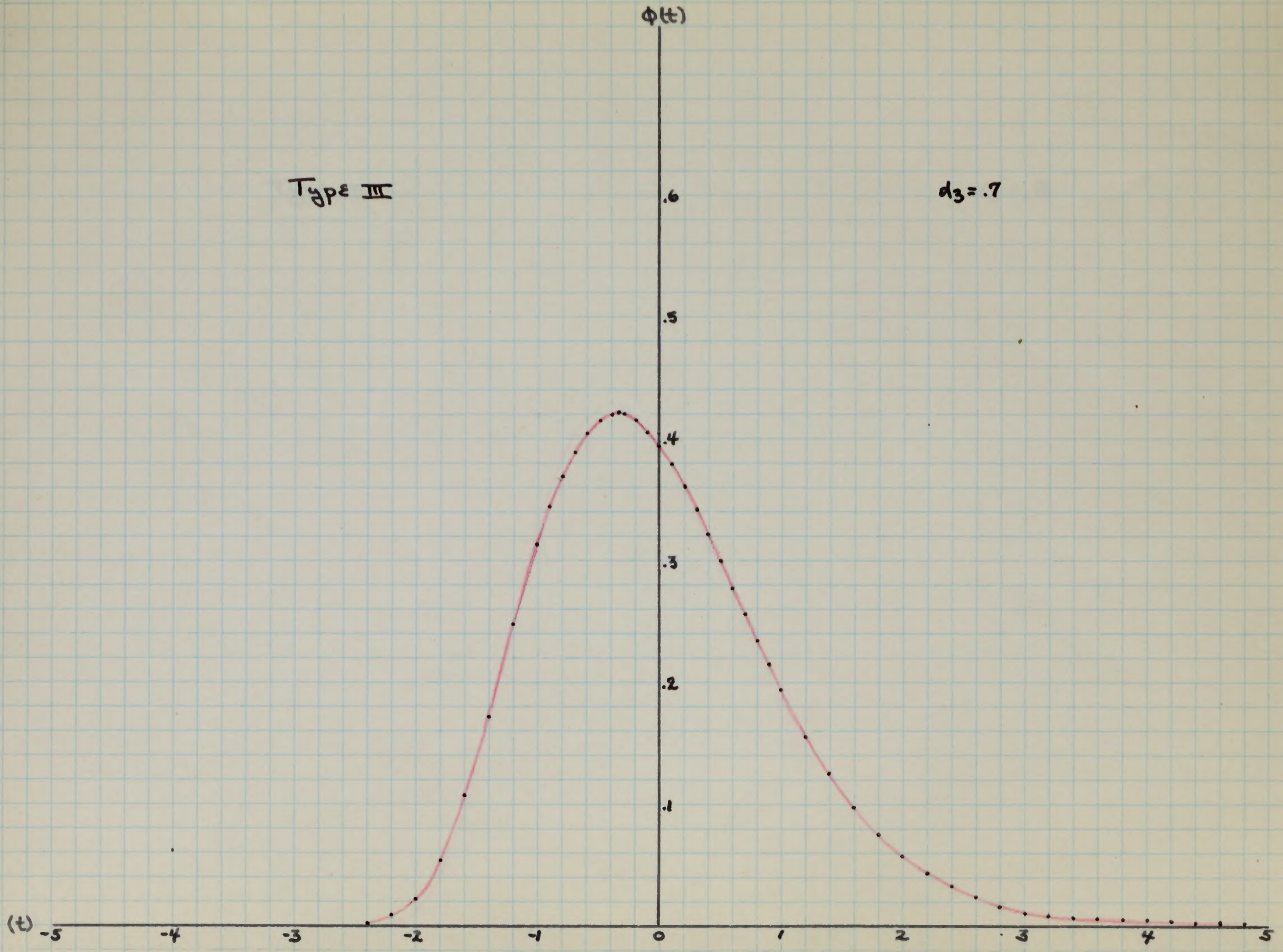
Type III

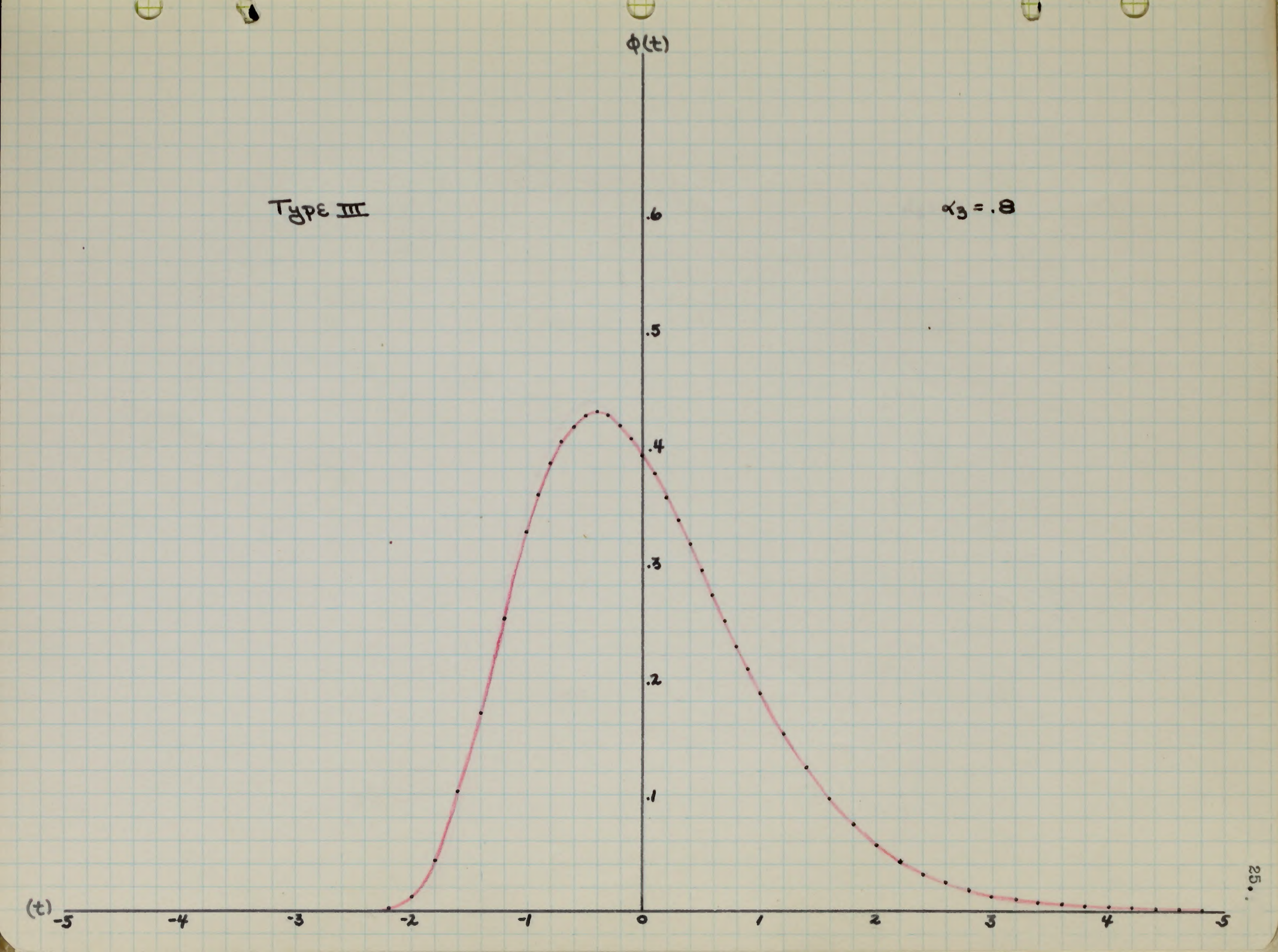
$\alpha_3 = .4$




Type III
$\alpha_3 = .5$
$\Phi(t)$
.6
.5
.4
.3
.2
.1
(t)
-5
-4
-3
-2
-1
0
1
2
3
4
5

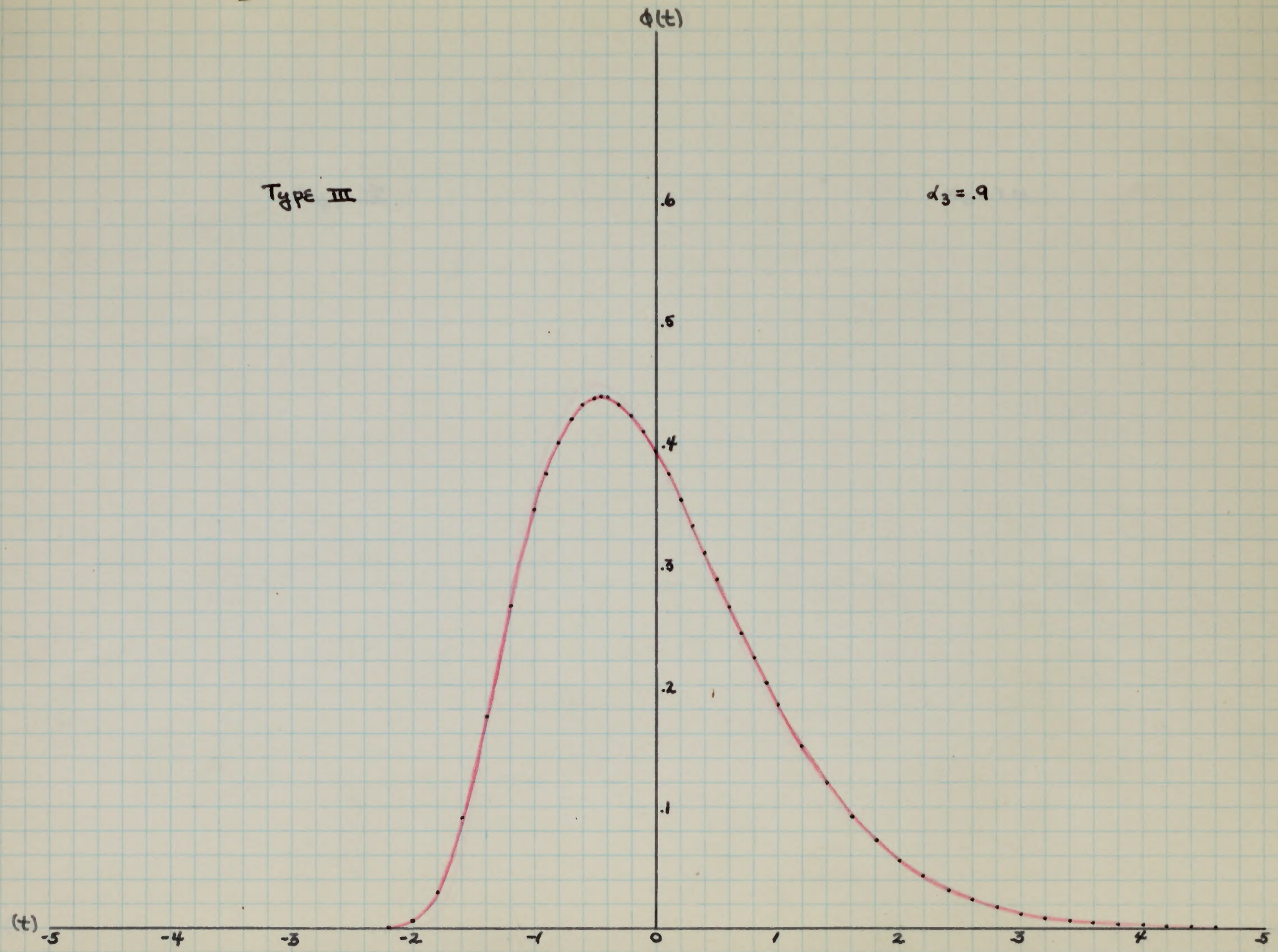

Type III
$\alpha_3 = .6$
$\Phi(t)$
.6
.5
.4
.3
.2
.1
(t)
-5
-4
-3
-2
-1
0
1
2
3
4
5


Type III
$d_3 = .7$
$\phi(t)$
.6
.5
.4
.3
.2
.1
(t)
-5
-4
-3
-2
-1
0
1
2
3
4
5


Type III
$\alpha_3 = .8$
$\phi(t)$
.6
.5
.4
.3
.2
.1
(t)
-5
-4
-3
-2
-1
0
1
2
3
4
5

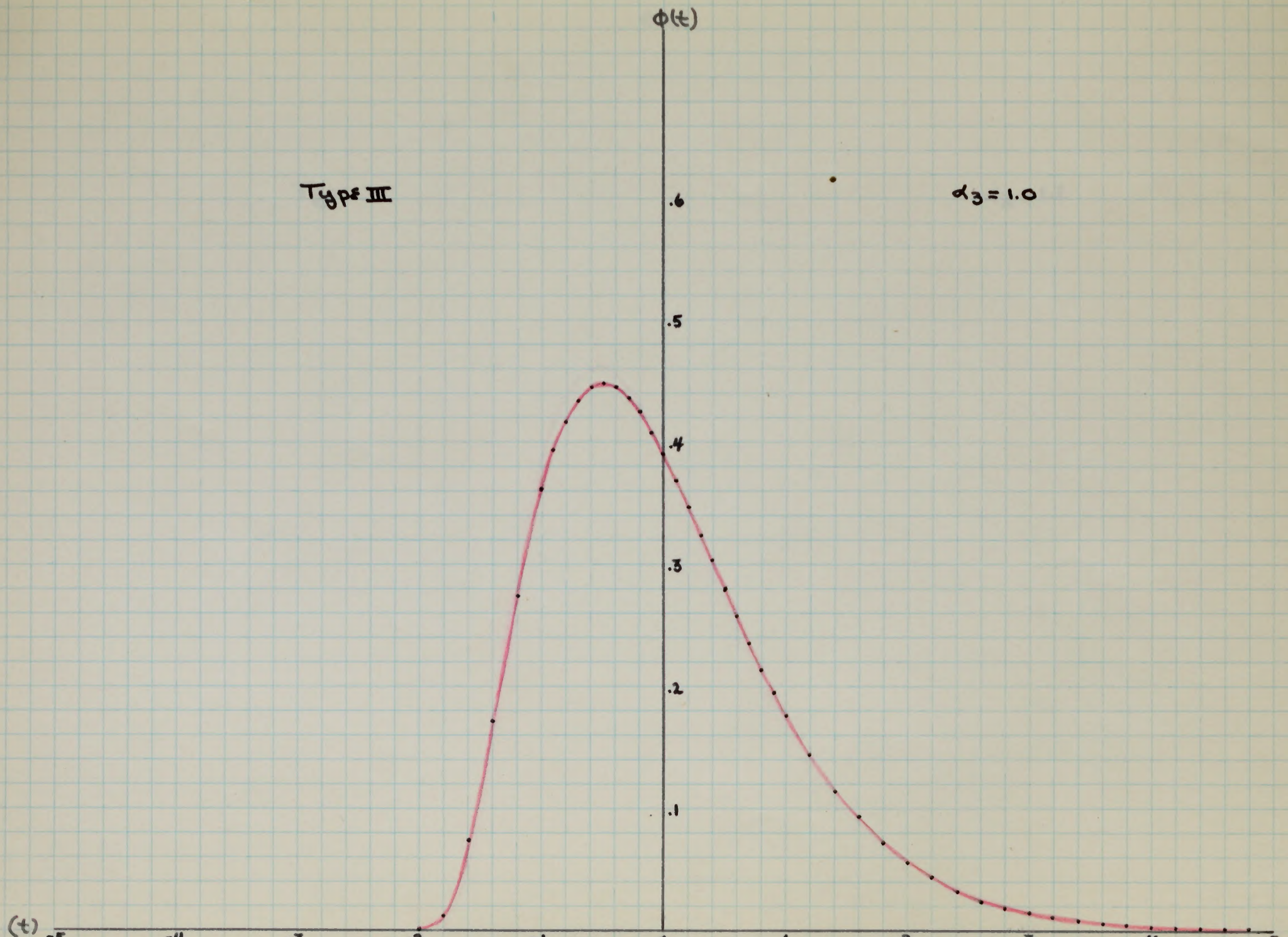
Type III

$\alpha_3 = .9$



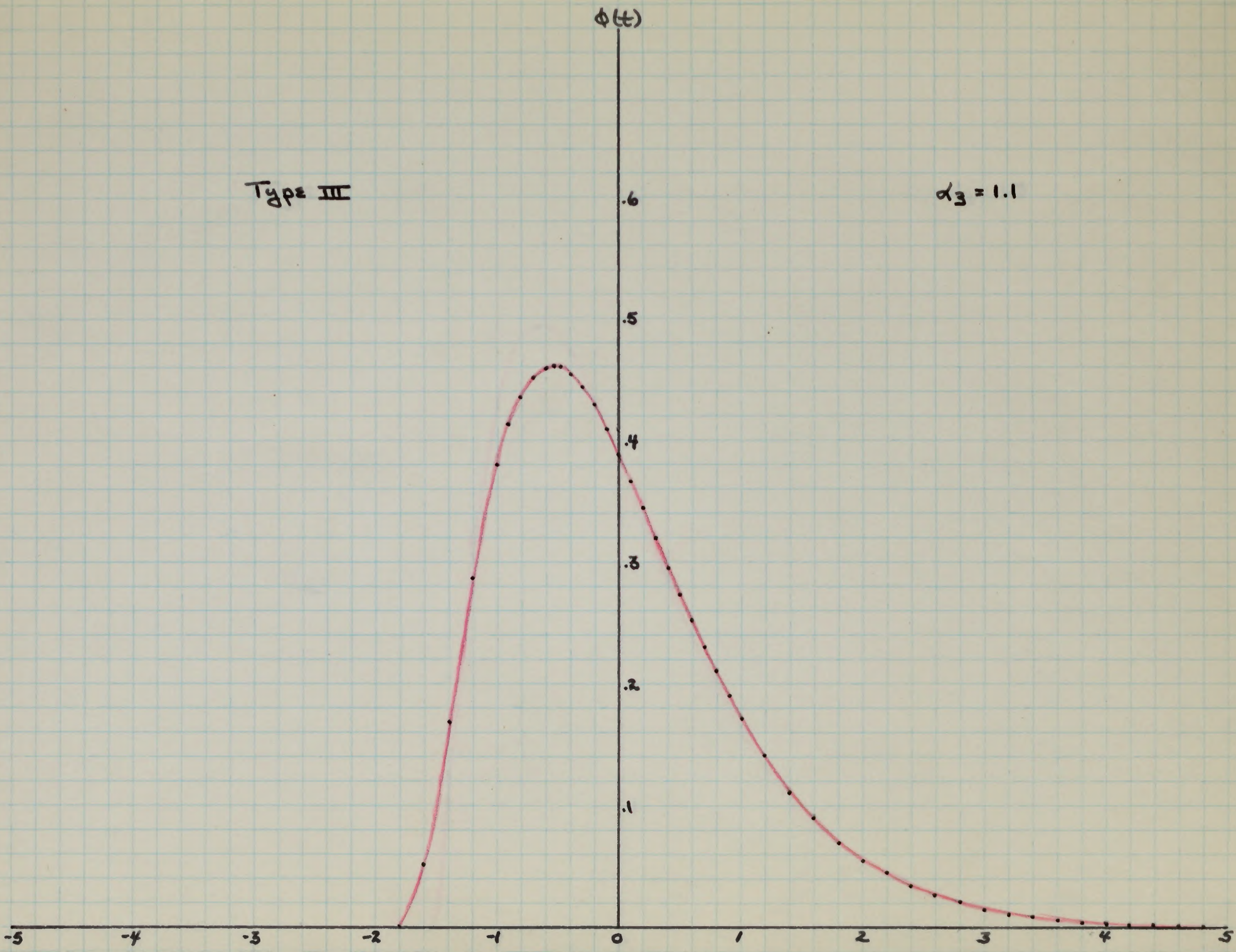
Type III

$\alpha_3 = 1.0$



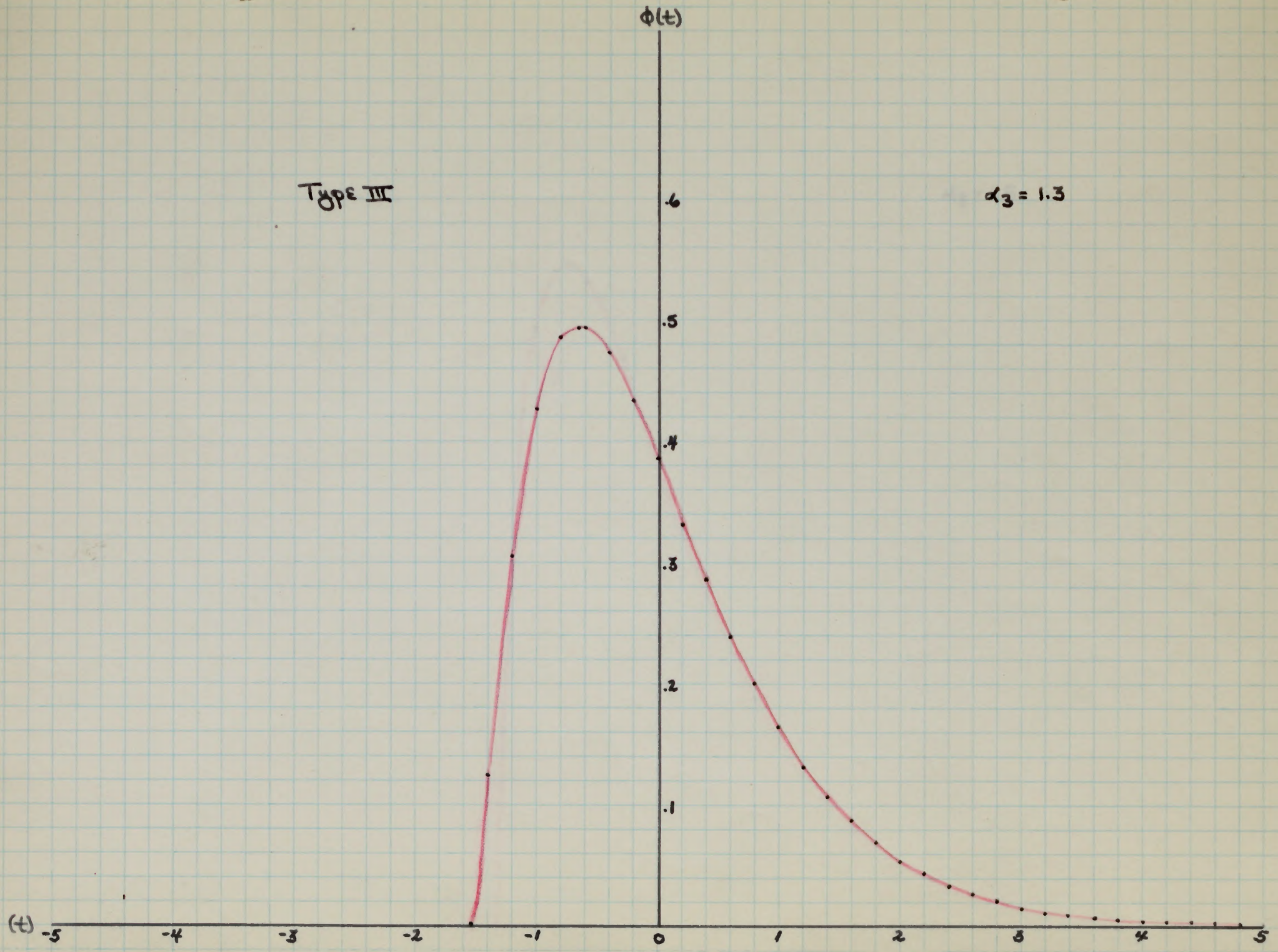
Type III

$\alpha_3 = 1.1$



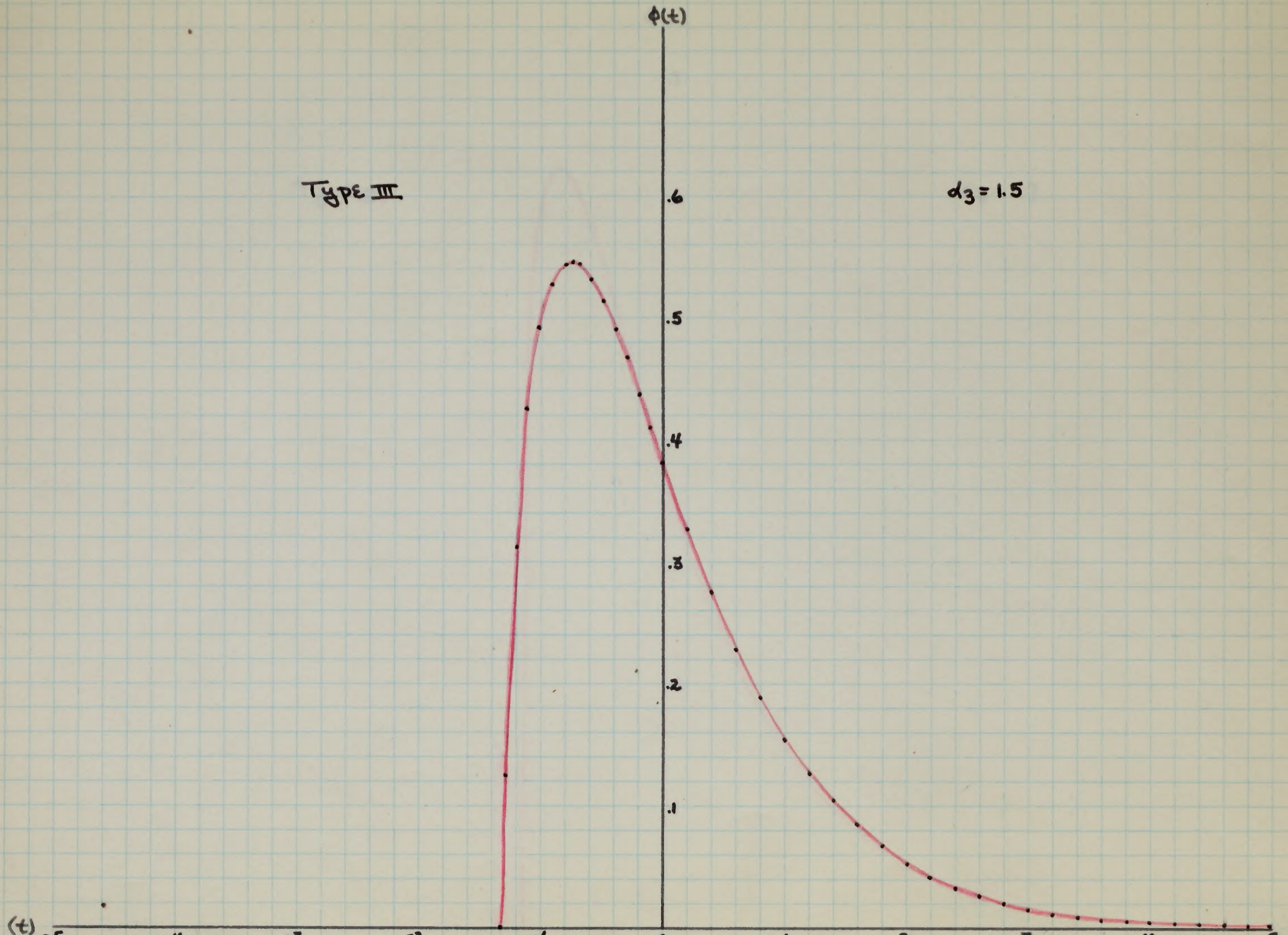
Type III

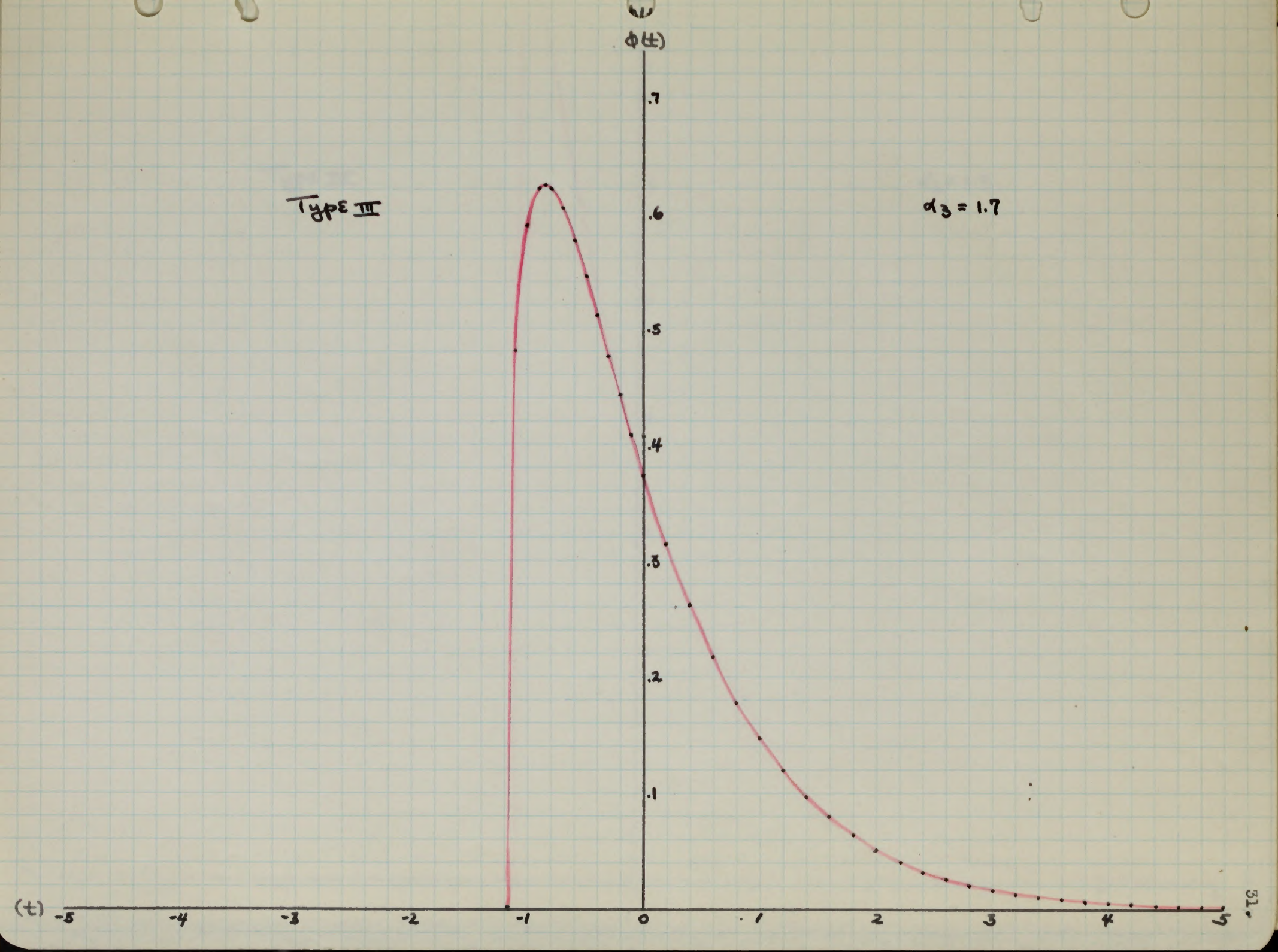
$\alpha_3 = 1.3$



Type III

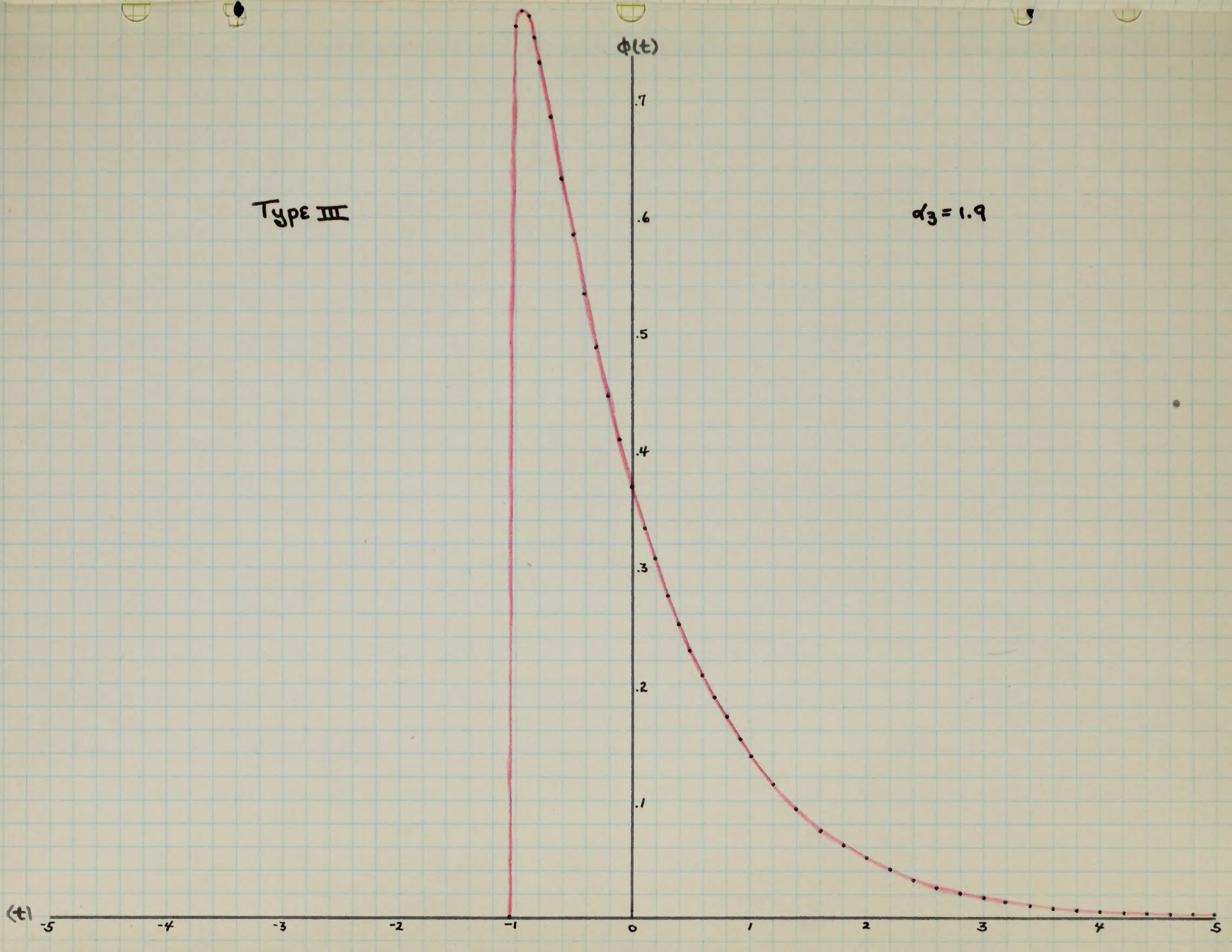
$\alpha_3 = 1.5$




Type III
$\alpha_3 = 1.7$
$\phi(t)$
.7
.6
.5
.4
.3
.2
.1
(t)
-5
-4
-3
-2
-1
0
1
2
3
4
5

Type III

$\alpha_3 = 1.9$



Type III

$\phi(t)$

.7
.6
.5
.4
.3
.2
.1

-5 -4 -3 -2 -1 1 2 3 4 5

(t)

The (α^2, δ) Chart For The PearsonSystem

of Frequency Curves.

(The subscript L refers to bell-shaped **surves.**)

Taken from Chart for Pearson System of Frequency Curves by Cecil C. Craig. Annals of Mathematical Statistics, March, 1936--Vol. Vll, Number 1.

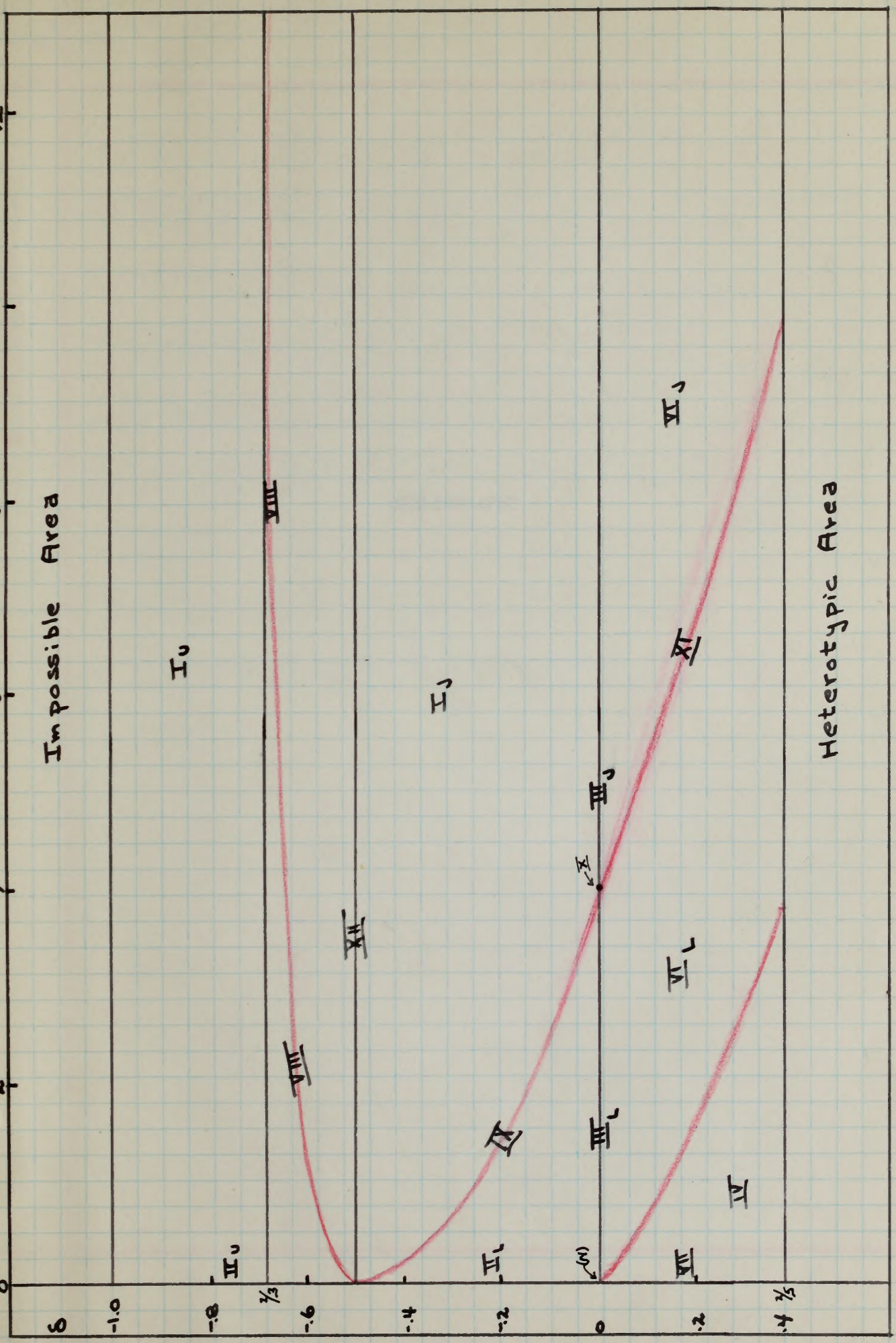
Considering the positions of the Type X, the Type 111, and the Normal curve on the (α^2, δ) Chart, we noted in each case that one of the conditions was that $\delta = 0$. Hence along the line $\delta = 0$ we should find the representations of the curves.

With both $\delta = 0$ and $\alpha_3 = 0$ we have a point representing the Normal Curve.

With $\delta = 0$ and $\alpha_3 = 2$ we have a point representing the Type X Curve.

With $\delta = 0$ and $\alpha_3 < 2$ we have a section of the line $\delta = 0$ representing the various Type 111 Curves that are Bell-shaped, with a skewness depending upon α_3.

With $\delta = 0$ and $\alpha_3 > 2$ we have a section of the line $\delta = 0$ representing the various Type 111 Curves that are J-shaped, with a skewness depending upon α_3.


$d_3^2 = \beta$
Impossible Area
Heterotypic Area
H^u
H^J
H^J
VIII
VIII
VII
VII
VI
VI
V
V
IV
IV
III
III
II
II
I
I
X
X
(N)
0
2
4
6
8
10
12
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
δ
⅓
⅕

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